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# DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/MANAGEMENT/ 

 COMMERCIAL PRACTICE, APRIL-2022
## ENGINEERING MATHEMATICS - I

[Maximum marks: 100]
(Time: 3 Hours)

## PART - A

Maximum marks : 10
I (Answer all the questions in one or two sentences. Each question carries 2 marks)

1. Prove that $\cos ^{2} A-\sin ^{2} A=1-2 \sin ^{2} A$.
2. If $\cos A=\frac{4}{5}$ and A is acute, find $\cos 3 A$
3. Find the area of the triangle ABC , given $b=3 \mathrm{~cm}, \mathrm{c}=2 \mathrm{~cm}, A=30^{\circ}$.
4. If $y=x \tan x$, Find $\frac{d y}{d x}$
5. Find the velocity and acceleration at time ' $t$ ' of a particle moving according to

$$
\mathrm{s}=t^{2}-3 t+1
$$

## PART - B

Maximum marks : 30
II (Answer any five of the following questions. Each question carries 6 marks)

1. Express $4 \cos \theta+3 \sin \theta$ in the form $R \sin (\theta+\alpha)$. Where $\alpha$ is acute.
2. Prove that $\cos 20 \cdot \cos 40 \cdot \cos 80=\frac{1}{8}$
3. Prove that in any triangle $\mathrm{ABC}, R\left(a^{2}+b^{2}+c^{2}\right)=a b c(\cot A+\cot B+\cot C)$
4. Differentiate $\cos x$ by the method of first principles.
5. Find $\frac{d y}{d x}$ if $x^{2}+y^{2}=25 x y$.
6. Find the equation to the tangent and normal to the curve $x^{2}+y^{2}=25$ at $(3,-4)$.
7. Prove that $\cos 570^{\circ} \sin 510^{\circ}-\sin 330^{\circ} \cos 390^{\circ}=0$.

PART - C
Maximum marks : 60
(Answer one full question from each unit. Each full question carries 15 marks)
UNIT -I
III. (a) Prove that $\frac{\cos \theta}{1+\sin \theta}+\frac{1+\sin \theta}{\cos \theta}=2 \sec \theta$
(b) If $\sin \mathrm{A}=\frac{8}{17}, \sin B=\frac{3}{5} ; \mathrm{A}, \mathrm{B}$ are acute, find $\sin (A-B)$ and $\cos (A+B)$
(c) From the top of a light house 90 m high, the angles of depression of two boats on the sea level are $45^{\circ}$ and $60^{\circ}$. Find the distance between the boats.

## OR

IV.(a) Prove that $\frac{\operatorname{cosec} A}{\operatorname{cosec} A-1}+\frac{\operatorname{cosec} A}{\operatorname{cosec} A+1}=2 \sec ^{2} A$
(b) If $\sin \mathrm{A}=\frac{2}{5}$ and $A$ is acute, find $\sin 2 A$ and $\cos 2 A$.
(c) Show that $\tan 75^{\circ}+\cot 75^{\circ}=4$ without using tables.

## UNIT-II

V. (a) Prove that $\frac{\sin A+\sin 3 A+\sin 5 A}{\cos A+\cos 3 A+\cos 5 A}=\tan 3 A$
(b) Prove that $\sin \theta+\sin 3 \theta+\sin 5 \theta+\sin 7 \theta=4 \cos \theta \cdot \cos 2 \theta \cdot \sin 4 \theta$.
(c) Solve $\triangle A B C$, given $a=2 \mathrm{~cm}, b=3 \mathrm{~cm}$ and $c=4 \mathrm{~cm}$.

## OR

VI. (a) Prove that $\sin 50^{\circ}-\sin 70^{\circ}+\sin 10^{\circ}=0$.
(b) Prove that $\cos 55^{\circ}+\cos 65^{\circ}+\cos 175^{\circ}=0$
(c) Solve $\triangle A B C$, given $a=5 \mathrm{~cm}, b=8 \mathrm{~cm}$ and $C=30^{\circ}$.

## UNIT-III

VII. (a) Evaluate (i) $\lim _{x \rightarrow 0} \frac{\sin 3 x \cdot \cos x}{x}$ (ii) $\lim _{x \rightarrow \infty} \frac{3 x^{2}-x+1}{2 x^{2}+2 x-1}$
(b) If $x=\mathrm{a} \sec \theta ; y=\mathrm{b} \tan \theta$, find $\frac{d y}{d x}$
(c) If $\mathrm{y}=A e^{n x}+B e^{-n x}, A, B$ are constants, show that $\frac{d^{2} y}{d x^{2}}-\mathrm{n}^{2} \mathrm{y}=0$.

## OR

VIII.(a) Evaluate (i) $\lim _{x \rightarrow 0} \frac{1-\cos x}{x^{2}}$ (ii) $\lim _{x \rightarrow 1} \frac{x^{3}-1}{x-1}$
(b) If $y=\log (\sec x-\tan x)$, show that $\frac{d y}{d x}=-\sec x$.
(c) If $y=a \sin m x$, Prove that $\frac{d^{2} y}{d x^{2}}+m^{2} y=0$

## UNIT-IV

IX. (a) Find the equations of tangent and normal to the curve $y=3 x^{2}+x+2$ at (1,2).
(b) A circular patch of oil spreads out on water, the area is growing at the rate of $6 \mathrm{~cm}^{2}$ per minute. How fast is the radius increasing when the radius is 2 cms ?
(c) Prove that a rectangle of fixed perimeter has its maximum area when it becomes a square

## OR

X. (a) The distance travelled by a particle moving along a straight line is given by $S=2 t^{3}-9 t^{2}+12 t+6$. Find the value of ' $t$ ' when the acceleration is zero.
(b) The radius of a circular plate is increasing in length at $0.1 \mathrm{~cm} / \mathrm{sec}$ when heated. What is the rate at which the area is increasing when the radius is 12 cm .
(c) The deflection of a beam is given by $y=2 x^{3}-9 x^{2}+12 x$, find the maximum deflection.

