

**DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/
MANAGEMENT/COMMERCIAL PRACTICE, APRIL – 2021**

ENGINEERING MATHEMATICS – I

[Maximum Marks: 75]

[Time: 2.15 Hours]

PART-A

(Answer **any three** questions in one or two sentences. Each question carries 2 marks)

- I. 1. Evaluate $\sin 30 + \cos 60 - \tan^2 45$.
2. If $\tan \theta = \frac{1}{2}$, find $\tan 2\theta$?
3. Find the area of a triangle having the sides $a = 4\text{cm}$, $b = 2\text{cm}$ and the included angle $C = 30^\circ$.
4. Find $\lim_{\theta \rightarrow 0} \frac{\sin 5\theta}{\theta}$.
5. Find the slope of the tangent to the curve $y = \tan x$ at $x = \frac{\pi}{4}$. (3 x 2 = 6)

PART-B

(Answer **any four** of the following questions. Each question carries 6 marks)

- II. 1. From the top of a light house 90m high, the angles of depression of two boats on the sea level are 45° and 60° . Find the distance between the boats.
2. Express $\sin x - \sqrt{3} \cos x$ in the form $K \sin(x - \alpha)$.
3. Prove that $\cos 20 \cos 40 \cos 80 = 1/8$
4. Prove that in a ΔABC , $(a + b) \sin \frac{C}{2} = c \cos \left(\frac{A-B}{2}\right)$
5. Differentiate ' $\sin x$ ' by the method of first principles.
6. Find $\frac{dy}{dx}$ if $2x^3 + 6xy + 2y^3 = 16$.
7. The deflection of a beam is given by $y = 4x^3 + 9x^2 - 12x + 5$. Find the maximum deflection. (4 x 6 = 24)

PART-C

(Answer **any of the three units** from the following. Each full question carries 15 marks)

UNIT – I

- III. (a) Prove that $\frac{\sin \theta}{1 + \cos \theta} + \frac{1 + \cos \theta}{\sin \theta} = 2 \operatorname{cosec} \theta$ (5)

(b) Prove that $\cos 120 \sin 210 - \sin 240 \cos 330 = 1$. (5)

(c) Find the value of $\tan 75$ without using tables and use it show that $\tan 75 + \cot 75 = 4$. (5)

OR

IV. (a) Prove that $\sqrt{\frac{1-\sin\theta}{1+\sin\theta}} + \sec\theta - \tan\theta$. (5)

(b) If $\tan \theta = \frac{5}{12}$, θ lies in the third quadrant, find all other trigonometric functions. (5)

(b) Prove that $\sin(A + B) \sin(A - B) = \cos^2 B - \cos^2 A$. (5)

UNIT – II

V. (a) Prove that $\frac{\sin 3x}{\sin x} - \frac{\cos 3x}{\cos x} = 2$ (5)

(b) Prove that $\frac{\cos 3A - \cos A}{\sin A - \sin 3A} = \tan 2A$. (5)

(c) Solve ΔABC , if $a = 2, b = 3, C = 4$. (5)

OR

VI. (a) Prove that $\operatorname{Cosec} 2A + \cot 2A = \cot A$. (5)

(b) Prove that $\cos 55 + \cos 65 + \cos 175 = 0$ (5)

(c) Two angles of triangular plot of land are 53° and 67° and the side between them is measured to be 100m. How many meters of fencing is required to fence the plot? (5)

UNIT- III

VII. (a) Find i) $\lim_{x \rightarrow 3} \frac{x^3 - 27}{x^2 - 9}$ ii) $\lim_{x \rightarrow \infty} \frac{x^2 + x - 1}{2x^2 + 3x + 1}$ (6)

(b) Use quotient rule to find the derivative of $\tan x$. (4)

(c) If $y = x + \frac{1}{x}$, then prove that $x^2 y'' + xy' = y$. (5)

OR

VIII. (a) Find $\frac{dy}{dx}$ if i) $y = e^{2x} \log 2x$ ii) $y = \sin^5(x^2)$ (6)

(b) Find $\frac{dy}{dx}$ if $x = a \sec \theta, y = b \tan \theta$. (4)

(c) If $y = a \cos mx + b \sin mx$ then show that $y'' + m^2 y = 0$. (5)

UNIT - IV

IX. (a) The distance travelled by a particle moving along a straight line after time t is given by

$s = 2t^3 - 9t^2 + 12t + 6$. Find the value of t when the acceleration is zero. (5)

(b) Find the equation of the tangent and normal to the curve $y = x^2 + x - 1$ at $x = 2$. (5)

(c) Prove that a rectangle of fixed perimeter has its maximum area when it becomes a square. (5)

OR

- X. (a) Find the values of x for which the tangent to the curve $y = \frac{x}{x^2 + 1}$ will be parallel to the x axis. (5)
- (b) Air is pumped into a spherical rubber bladder of radius 3 inches. If the radius increase at a uniform rate of 1 inch per minute, find the rate at which the volume is increasing at the end of 3 minutes. (5)
- (c) The bending moment of a rod of length 10 m and weighing 40kg and resting at its ends at a distance of x m from one end is given by $M = 2(10 - x^2)$. Find the maximum bending moment. (5)
