

**DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/
MANAGEMENT/COMMERCIAL PRACTICE, NOVEMBER – 2020**

TECHNICAL MATHEMATICS - II

[Maximum Marks: 100]

[Time: 3 Hours]

PART-A

[Maximum Marks: 10]

(Answer **all** questions in one or two sentences. Each question carries 2 marks)

- I. 1. Evaluate $\lim_{x \rightarrow \infty} \frac{x^3 - 3x + 5}{2x^3 - 4x - 6}$
2. Find $\frac{dy}{dx}$ if $y = \cos\sqrt{x}$
3. Find the slope of the tangent to the curve $y = 3x^2 + x - 2$ at (1,2)
4. Evaluate $\int \sec(2x + 3) \tan(2x + 3) dx$.
5. Solve $\frac{d^2y}{dx^2} = \sin x$. (5 x 2 = 10)

PART-B

[Maximum Marks: 30]

(Answer any **five** of the following questions. Each question carries 6 marks)

- II 1. (a) Find 'a' if

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x \neq 0 \\ a, & x = 0 \end{cases}$$

is continuous at $x = 0$.

- (b) Using the method of first principles find the derivative of $y = x^2$

2. Find $\frac{dy}{dx}$ if
- (a) $x = a \cos^3 \theta, y = b \sin^3 \theta$. (b) $x^2 + xy + y^2 = 0$.
3. Prove that a rectangle of fixed perimeter has its maximum area when it is a square.
4. Find (a) $\int \frac{3 \cos x + 4}{\sin^2 x} dx$. (b) $\int \frac{x^2}{(8+x^3)^4} dx$
5. Evaluate $\int_0^{\frac{\pi}{2}} \sin 3x \cos x dx$.
6. Find the area between the curves $x^2 = 4y$ and $y^2 = 4x$.
7. Solve $(1+x) \frac{dy}{dx} - y = (1+x)^2$. (5 x 6 = 30)

PART-C

[Maximum Marks: 60]

(Answer **one** full question from each Unit. Each full question carries 15 marks)

UNIT – I

III (a) Evaluate $\lim_{\theta \rightarrow 0} \frac{\tan 3\theta + \sin 5\theta}{8\theta}$. (4)

(b) Find $\frac{dy}{dx}$ if (i) $y = x^2 \tan^{-1} x$ (ii) $y = \frac{e^{2x}}{x + \log x}$ (3+4)

(c) Find $\frac{d^2y}{dx^2}$ if $y = \frac{x}{4} + \frac{4}{x}$

OR

IV (a) Evaluate (i) $\lim_{x \rightarrow -1} \frac{x^3 + 1}{x + 1}$ (ii) $\lim_{x \rightarrow 1} \frac{x^2 + 4x - 5}{x^2 + x - 2}$. (2+3)

(b) If $y = \log(x + \sqrt{1 + x^2})$, show that $\frac{dy}{dx} = \frac{1}{\sqrt{1 + x^2}}$. (5)

(c) If $xy = ax^2 + \frac{b}{x}$, prove that $x^2 \frac{d^2y}{dx^2} + 2\left(x \frac{dy}{dx} - y\right) = 0$ (5)

UNIT – II

V (a) Find equation of tangent and normal to the curve $y = \sqrt{25 - x^2}$ at (4,3). (5)

(b) A spherical balloon is inflated with air such that its volume increases at the rate of 5cc per second. Find the rate at which its curved surface is increasing when its radius is 7cm. (5)

(c) Find the maximum value of the function $f(x) = 2x^3 - 9x^2 + 12x + 1$. (5)

OR

VI (a) Find the values of x for which the tangent to the curve $y = \frac{1+x}{(1-x)^2}$ will be parallel to

(i) x-axis (ii) y-axis (6)

(b) A particle projected vertically upwards has height 'h' at time 't' given by the equation $h = 60t - 15t^2$. Find the greatest height attained. (4)

(c) Water is running out of a conical funnel at the rate of 1 cubic inch per second. If the radius of the base of the funnel is 4 inches and the altitude is 8 inches find the rate at which the water level is dropping when its depth is 6 inches. (5)

UNIT- III

- VII (a) Find $\int \sin^3 x \, dx$. (3)
- (b) Find $\int \frac{1}{\sqrt{3x+4}} \, dx$. (3)
- (c) Find $\int (\tan x + \cot x)^2 \, dx$ (3)
- (d) Find $\int \log x \, dx$ (3)
- (e) Find $\int x e^{-x} \, dx$. (3)

OR

- VIII (a) Evaluate $\int_0^{\pi} \frac{1-\sin x}{x+\cos x} \, dx$. (5)
- (b) Evaluate $\int_0^{\frac{3\pi}{2}} x \cos 3x \, dx$. (5)
- (c) Evaluate $\int_1^2 \frac{x^2+1}{(x^3+3x)^2} \, dx$. (5)

UNIT - IV

- IX (a) Obtain the area enclosed between the line $2x + y = 1$ and the curve
 $y = x^2 - 6x + 4$. (5)
- (b) Find the volume of the ellipsoid when the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ is rotated about
the x-axis (5)
- (c) Solve $\frac{dy}{dx} + 2y \tan x = \sin x$ (5)

OR

- X (a) Find area of a circle of radius 'a' using integration. (5)
- (b) Find the volume obtained by rotating one arch of the curve $y = 2 \sin 3x$, about
the x-axis. (5)
- (c) Solve: $3e^x \tan y \, dx + (1 - e^x) \sec^2 y \, dy = 0$. (5)
