

**DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/
MANAGEMENT/COMMERCIAL PRACTICE, NOVEMBER – 2024**

ENGINEERING MATHEMATICS I

(Maximum Marks:100)

(Time: 3 Hours)

PART - A

(Maximum Marks : 10)

Marks

I. Answer **all** the questions. Each question carries 2 marks.

1. Evaluate $\lim_{x \rightarrow 2} \frac{x\sqrt{x} - 2\sqrt{2}}{x-2}$

2. Prove that $(\sin A + \cos A)^2 = 1 + 2 \sin A \cos A$.

3. Find the rate of change of volume of a cube w.r.t the side x.

4. Find the second derivative $\sin x \cdot \cos x$.

5. If $\tan A = 0.38$, find $\tan 2A$.

(5 x 2 = 10)

PART - B

(Maximum Marks: 30)

II Answer **any five** questions from the following. Each question carries 6 marks.

1. Derivative of $\cos x$ w.r.t x using first principle.

2. A particle moves such that the displacement from a fixed-point O is given by $S = a.e^{2t} + b.e^{-2t}$ where a and b are constant. Prove that the acceleration varies as its displacement.

3. The deflection of a beam is given by $y = 2x^3 - 9x^2 + 12x$. find the maximum deflection.

4. Express $4\cos x + 3 \sin x$ in the form $R\sin(x + a)$.

5. Prove that $\sin\theta + \sin 3\theta + \sin 5\theta + \sin 7\theta = 4\cos \theta \cdot \cos 2\theta \cdot \sin 4\theta$.

6. Prove that $R(a^2 + b^2 + c^2) = abc (\cot A + \cot B + \cot C)$

7. Prove that (i) $\sin 78^\circ - \sin 18^\circ + \cos 132^\circ = 0$

(ii) $\frac{\sin A - \sin B}{\cos A + \cos B} = \tan \frac{A-B}{2}$

(5 x 6 = 30)

PART – C

(Maximum Marks: 60)

(Answer **one full** question from each unit. Each full question carries 15 marks.)

UNIT - I

- III (a) Prove that $\frac{\cos A - \sin A}{\cos A + \sin A} = \tan(45 - A)$ (5)
- (b) Find the value of $\tan 5$ without using tables and show that $\tan 75 + \cot 75 = 4$ (5)
- (c) If $\tan A = -\frac{3}{4}$, $\sin B = \frac{5}{13}$ (A lies in fourth quadrant and B lies in second quadrant) (5)

OR

- IV (a) Prove that $\sin 240 \cdot \cos 330 - \cos 120 \cdot \sin 210 = -1$. (5)
- (b) Prove that $(\cot A - 1)^2 + (\cot A + 1)^2 = 2 \operatorname{cosec}^2 A$. (5)
- (c) Prove that $\frac{1 + \sin \theta}{\cos \theta} + \frac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$. (5)

UNIT – II

- V (a) Prove that $\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$ (5)
- (b) Prove that $2(b \cos A + c \cos B + a \cos C) = a^2 + b^2 + c^2$ (5)
- (c) Solve triangle ABC given $a = 2$ cm, $b = 3$ cm, $c = 4$ cm (5)

OR

- VI (a) Prove that $\sin 10 \cdot \sin 50 \cdot \sin 70 = \frac{1}{8}$. (5)
- (b) Solve triangle ABC $b = 5$ cm, $c = 8$ cm, $A = 30^\circ$. (5)
- (c) Prove that $\frac{\sin 3A}{\sin A} + \frac{\cos 3A}{\cos A} = 4 \cos 2A$. (5)

UNIT – III

- VII (a) If $x = \theta + \sin \theta$, $y = 1 - \cos \theta$, then prove that $\frac{dy}{dx} = \tan \frac{\theta}{2}$ (5)
- (b) If $y = a \cos nx + b \sin nx$, show that $y'' + n^2 y = 0$ (5)
- (c) Find the derivative of $\frac{x + \sqrt{x}}{1 + \sqrt{x}}$. (5)

OR

- VIII (a) Find $\lim_{x \rightarrow \pi/2} \frac{\cos x}{\frac{\pi}{2} - x}$ (5)
- (b) Find the derivative of $\frac{\cos 3x}{e^{3x}}$ (5)
- (c) Find the derivative of $x^2 + y^2 + 2gx + 2fy + c = 0$. (5)

UNIT – IV

- IX (a) Prove that the rectangle of fixed perimeter has its maximum area when it becomes a square. (5)
- (b) Find the turning point of x^2-3x^2-9x+5 (5)
- (c) A spherical balloon is inflated with air such that its volume increases at the rate of 10 c.c/s. Find the rate at which curved surface is increasing when its radius is 15 cm. (5)

OR

- X (a) A stone is dropped in to still water, the radius of the outermost ripple then found Increases at the rate of 10 cm/s. How fast is the area increasing when the radius is 24cm. (5)
- (b) Find the equation to the tangent and normal to the curve $y=\cos x$ at $x = \pi/6$. (5)
- (c) If S denotes the displacement of a particle at a time t sec. and $S = t^3-6t^2+8t-4$. Find the time when the acceleration is 12 cm/sec². (5)
