

**DIPLOMA EXAMINATION IN ENGINEERING/TECHNOLOGY/
MANAGEMENT/COMMERCIAL PRACTICE, APRIL – 2024**

ENGINEERING MATHEMATICS – I

[Maximum Marks: **100**]

[Time: **3 Hours**]

PART-A

[Maximum Marks: **10**]

I. (Answer *all* questions in one or two sentences. Each question carries **2** marks)

1. Prove that $(\sin \theta + \cos \theta)^2 = 1 + \sin 2\theta$.
2. Prove that $\sin 60 \cos 30 + \cos 60 \sin 30 = 1$
3. Find the area of the triangle, given $a = 4\text{cm}, b = 2\text{cm}$ and $C = 30^\circ$
4. Find the derivative of $x^2 \log x$
5. Find the slope of the line $y = x^2 - 3x + 2$ at the point $(3,2)$ (5 x 2 = 10)

PART-B

[Maximum Marks: **30**]

II. (Answer any *five* of the following questions. Each question carries **6** marks)

1. Express $\sqrt{3}\sin x + \cos x$ in the form of $R\sin(x + \alpha)$ where α is acute.
2. Prove that $\sin 20^\circ \sin 40^\circ \sin 80^\circ = \frac{3}{16}$
3. If $y = a\cos(\log x) + b\sin(\log x)$, show that $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$
4. An open box is to be made out of a square sheet side 18cm, by cutting off equal squares at each corner and turning up sides. What size of the squares should be cut in order that the volume of the box may be maximum?
5. In ΔABC , $a = 4\text{cm}, b = 5\text{cm}, c = 7\text{cm}$. Solve ΔABC .
6. A person standing on the bank of a river observes that the angle of elevation of the top of a tree standing on the opposite bank is 60° . When he moves 40 meters away from the bank, he find the angle of elevation to be 30° . Find the height of the tree and the width of the river.
7. Differentiate $\cos x$ by the method of first principles. (5 x 6 = 30)

PART-C

[Maximum Marks: 60]

(Answer **one** full question from each Unit. Each full question carries **15** marks)

UNIT - I

- III. (a) Prove that $\frac{\sec \theta}{\sec \theta + 1} + \frac{\sec \theta}{\sec \theta - 1} = 2 \operatorname{cosec}^2 \theta$ (5)
- (b) Prove that $\frac{\tan 45^\circ - \tan 30^\circ}{1 + \tan 45^\circ \tan 30^\circ} = 2 - \sqrt{3}$ (5)
- (c) From the top of a light house 90m high, the angles of depression of two boats on the sea level are 45° and 60° . Find the distance between the boats. (5)

OR

- IV. (a) Prove that $\sqrt{\frac{1 - \sin \theta}{1 + \sin \theta}} = \sec \theta - \tan \theta$. (5)
- (b) If $\tan A = \frac{1}{2}$, $\tan B = \frac{1}{3}$, Prove that $A + B = 45^\circ$. (5)
- (c) If $\cos \theta = -\frac{1}{2}$, $\pi < \theta < \frac{3\pi}{2}$, find the value of $4 \tan^2 \theta - 3 \operatorname{cosec}^2 \theta$ (5)

UNIT - II

- V. (a) Prove that $\cos^4 A - \sin^4 A = \cos 2A$ (5)
- (b) Prove that $2 \tan 10^\circ + \tan 40^\circ = \tan 50^\circ$ (5)
- (c) Two angles of a triangular plot of land are $53^\circ 17'$ and $67^\circ 09'$ and the side between them is measured to be 100cm. How many meters of fencing is required to fence the plot. (5)

OR

- VI. (a) Prove that $\frac{\sin A + \sin 3A + \sin 5A}{\cos A + \cos 3A + \cos 5A} = \tan 3A$ (5)
- (b) Prove that $\cos 3A + \cos 5A + \cos 9A + \cos 17A = 4 \cos 4A \cdot \cos 6A \cdot \cos 7A$ (5)
- (c) Prove that $R(a^2 + b^2 + c^2) = abc(\cot A + \cot B + \cot C)$ (5)

UNIT- III

- VII. (a) Prove that $\lim_{x \rightarrow c} \frac{x^n - c^n}{x - c} = nc^{n-1}$. (5)
- (b) If $x = a(\cos t + t \sin t)$, $y = b(\sin t - t \cos t)$, find $\frac{dy}{dx}$ (5)
- (c) Find $\frac{dy}{dx}$, if $x^2 + xy + y^2 = 0$ (5)

OR

VIII. (a) Differentiate the following with respect to x

(i) $e^{(\cos x + 2x^2)}$ (ii) $x^2(1 + \cos x)$ (2+3=5)

(b) If $x = a \sec \theta, y = b \tan \theta$, find $\frac{dy}{dx}$ (5)

(c) If $y = x + \frac{1}{x}$, prove that $x^2 y'' + xy' = y$ (5)

UNIT - IV

IX. (a) Find the velocity and acceleration of a body whose displacement is given by the

equation $s = \frac{2}{3}t + cost$ (5)

(b) A balloon is spherical in shape. Gas is escaping from it at the rate of 10 cc/sec.
How fast is the surface area shrinking when the radius is 15 cm. (5)

(c) Find minimum values of $2x^3 - 9x^2 + 12x + 2$. (5)

OR

X. (a) A circular plate of radius 3 inches expands when heated at the rate of 2 inch per sec.

Find the rate at which the area of the plate is increasing at the end of 3 secs. (5)

(b) For what values of x is the tangent to the curve $\frac{x}{(1-x)^2}$ parallel to the x-axis? (5)

(c) The perimeter of a rectangle is 100m. Find the sides when the area is maximum. (5)
